Application of Linear Programming to Game Theory in Finance

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Abstract
The study examines the application of linear programming to game theory in finance using how the Telecommunication firms determine the optimal strategy from television and radio advertisements given that each company seeks to gain the largest market share. The methodology used involved a Game Theoretic approach and linear programming. The result of the analysis showed that for MTN to maintain its position as the leader in the industry, it should invest 0% of its M billion Naira in Television Advertisement and 100% in Radio Advertisement when competing with Airtel, Globacom and 9Naija. This will yield a pay-off of ₦9M billion. Findings revealed that for Globacom to gain the largest market share in the industry, it should invest 0% of her ₦N in Television Advertisement and 100% in Radio Advertisement when competing with MTN. This will produce a benefit of ₦9 billion Naira. The results also showed the optimal resource allocations for Airtel and Globacom for each firm to gain the largest market share in the industry when competing with MTN. Findings also revealed the pay-offs for both firms when they compete with MTN. It is therefore recommended that Nigerian Telecommunication firms use Game Theory in optimizing resource allocation between strategies.

Keywords: Linear Programming, Game theory Telecommunication, Strategies, MTN, Globacom, Airtel, 9Naija

1. INTRODUCTION
Organizations are set up to achieve goals and objectives. In trying to achieve the goals and objectives, decisions are to be made and resources have to be deployed. Some of the techniques that can assist managers of business organizations in operation research are game theory and linear programming. It assists managers in making effective decisions
in order to achieve efficiency, increase productivity and overall corporate performance (INFORMS, 2005).

In a competitive market, a producer or seller (party) always need to weigh the expected responses of their rivals. The issue is the approach adopted by each party to secure some competitive advantage. Mentioned below are some approaches that are currently used in underdeveloped countries like Nigeria. According to Osaze (1998), some approaches currently use in Nigeria for strategic management include hunch, intuitive and anticipatory, opportunistic, formal–structured, incrementing and adaptive.

Generally, allocation problems are concerned with the utilisation of limited resources to best advantage (Lucy, 2002). If there were no resource constraints, the organisation perhaps could allocate without optimising or optimise without considering the allocation implication but not both (Olayemi and Onyenweaku, 1999). Greater emphasis upon efficient utilisation of the existing resources and combination of enterprises in an optimal manner is paramount.

According to Akingbade, Luck and Patal (1991) linear programming in operation research is a problem-solving science-based activity using analysis and modelling as a basis for aiding decision-makers in organisations to improve the performance of the operations under their control. It is concerned with analysing complex problems and assisting decision-makers work out the best means of achieving objectives. It can be said to have been in existence since the beginning of mankind (Agbadudu, 2006). However, the concept emerged in 1940 during world war II, when a team of scientist was called upon by the military management in England to develop ways to make the most effective use of limited military resources (Anderson, Sweeney &Williams, 1997 and Taha, 2002). Their mission was to formulate specific proposals and plans for aiding the military commands to arrive at decisions on optimal utilisation of scarce military resources and also to implement the decisions effectively. The name “Operations Research” originated because the team was handling research on (military) operations. The United States military management took a leaf from the British military management and started the use of Operations Research.

Due to the successful utilisation of linear programming by military management in Britain and United States, managers of business organisations became interested in using the techniques to solve organisational problems. Consequently, in the early 1950’s, business organisations began to absorb some of the Operations Research men which trickled out of the military (Ekoko, 1999). Today, linear programming and game theory are dominant and indispensable decision making tools and are widely used in business organisations in western countries.

1.1. Focus of the Study

The study focuses on the Global System of Mobile Communication under Telecommunication industry in Nigeria. The study therefore limits its scope to MTN, Airtel, 9Naija and Globacom. The Telecommunications industry is undoubtedly a critical and strategic industry for any economy that desires to achieve economic growth; this is because of its positive contribution to the outputs growth of other sectors.
Telecommunication services are required for the smooth running of every firm in every industry. The sector also attracts foreign direct investment, thereby opening up the domestic economy to the global market. Although, the first use of telecommunication involved the use of a cable connection between the colonial office in London and Lagos in 1886, telephone services were later provided to government officials in 1893 and further later extended to Ilorin and Jebba. Also, a three channel line carrier system between Lagos and Ibadan was commissioned and later extended to Benin, Kano, Enugu, Kaduna and Osogbo, between 1946–1952, (Ajayi, 1999).

2. Literature Review

2.1. Linear Programming

According to Taha (2002) Linear programming is a technique for resolving problems of resource allocation. It is designed to assist management in its optimisation decisions involving the use of competing resources. It offers a simplified technique for specifying how to use limited resources or capacities of a business to obtain a particular objective such as least cost, least time or highest margin when these resources have alternative uses. There are situations where a business organization is faced with the problem of allocating its resources which include money, materials, land, machine time and labour time. It helps in the process of selecting the most desirable course of action from a number of available courses of action thereby giving management information for making effective decisions about the resources under its control. Its applicability is however restricted to problems that are entirely linear. The general formulation of a linear programming problem is given as:

\[
\text{Optimize } Z = \sum c_j x_j \\
\text{S.t } \sum a_{ij} x_j \leq b_i, \geq b_i, X_j \geq 0, i = 1,2,\ldots,m \text{ and } j = 1,2,\ldots,n
\]

Where:

- \(x_j\) = Decision variables
- \(c_j\) = Coefficients in the objective function
- \(Z\) = The objective to be optimized (either maximization or minimization)
- \(b_i\) = The set of constraints
- \(a_{ij}\) = The coefficients of the decision variables

Linear programming problems can be solved using the graphical method, the matrix method or the simple method (Spyros, 1999).

Given the dynamic nature of the environment, linear programming technique offers the type of flexibility management might require to cope with changing conditions. Results obtained through linear programming can be easily re-evaluated for changing conditions through sensitivity analysis (Ekanem & Iyoha, 2002).
2.2. Games Theory

A game is a description of the strategic static or dynamic interaction between opposing, or co-operating, interests (players) where the constraints and payoff for actions are taken into consideration and may or may not be known by the players before the game commences. On the other hand, a player is a basic entity in a game that is tasked with making choices for actions. A player can represent a person, machine, or group of persons within a game. Players can also represent lines of code and attack scripts. Player and firewalls strategically interact as they contend for network resources. Pay-off in an organisation can ranges from availability of bandwidth to more secured network environment which will result to better network performance and lower losses.

Game theory describes multi-person decision situations as games where each player chooses his/her actions which result in the best possible rewards for self, while anticipating the rational actions from other players. A player is the basic entity of a game that makes decisions and then performs actions. A game is a precise description of the strategic interaction that includes the constraints of, and payoffs for, actions that the players can take, but does not pay attention to the actions they actually take (Roy, Ellis, Shiva, Dasgupta, Shandilya & Wu, 2010).

There are four basic characteristics of a typical game as it applies to game theory. They include: Multiple player (two or more), Competitive in nature, Rules that guide every game and Payoffs for player.

Games theory provides a framework for analysing decision making among firms in a competitive situation. In the business world, many decisions are made in competitive situations where the decision of a competitor affects the decision of a firm. According to Camerer (2003) all situations in which at least one agent can only act to maximize his utility through anticipating (either consciously or just implicitly in his behaviour) the responses to his actions by one or more other agents is called a game. The purpose of the game is for each player to select the strategy that will result in the best possible payoff or outcome regardless of what the player’s opponent does. The term player is used to denote each firm, which takes part in games related to decision making. Each player in a game faces a choice among two or more possible strategies. A strategy is a predetermined programme of play that tells a firm what actions to take in response to every possible strategy its competitors might use. The best strategy for each player is known as the optimal strategy. When each player in the game adopts a single strategy as an optimal strategy, then pure strategy game exists. On the other hand, when the player adopts a mixture of strategies, then it is a mixed strategy game. A pure strategy game can be solved according to the minima decision criteria while a mixed strategy game can be solved using expected gain and loss method or linear programming (Ekoko, 1999).

2.4. Theoretical Background

Game Theory often referred to as the Science of Strategy explains how players make decisions in conflicts or competition. It was initially applied to examine the economic
behaviour of firms, markets and consumers. It has been widely applied to the behaviour of producers with few competitors. Game Theory enables firms to make appropriate choices and helps in understanding or predicting the behaviour of competitors. Modern game theory was introduced with the publication of the book ‘Theory of Games and Economic Behaviour’ authored by Morgenstern and Von Neumann (1953). Morgenstern and Von Neumann examined cooperative games involving various players in their book. It was later applied to other areas of specialization aside from Economics to social network formation, ethical behaviour, biology and other fields of specialization.

A game can be described as the mathematical representation of conflicts in which the players involved get results. It was originated by Antoine Augustin Cournot in 1838. The agents involved are usually called players (individuals or firms). Some of the components of a game include rules, pay-offs and strategies. Rules govern the conduct of the players; they are an important source of power in games. Strategies are courses of action taken to achieve the goals of the players. They influence the decision making process. In applying Game Theory to the competitive behaviour of firms, firms face a number of strategic decisions which can be taken to realize a desired pay-off. A pay-off is the outcome of adopting a strategy such as win, lose or draw often represented by a Pay-off Matrix. Firms can derive a range of pay-offs from the strategies they use. Such pay-offs can include greater profit for shareholders, increase in market share, improved chances of survival and eliminating a rival.

A game can be either Static or Dynamic. Static games involve players taking their decisions, without the knowledge of the actions of their competitors. Actions do not have to be taken at the same time, but rather it is as if the decisions are made simultaneously. Thus, the element of time is excluded from these kinds of games. Static games are described in strategic forms. The Prisoner’s Dilemma is an example of a static game wherein two suspects are arrested for a crime and are interrogated in separate cells. One of the suspects will be set free and the other will be imprisoned, if the first confesses and the other refuses to own up. If neither owns up, both of them will receive a lower punishment, than if they both confess. Each suspect is afraid the other will confess and thus is quick to confess to be released. In Dynamic games often represented in extensive forms, the players are able to know the actions of competitors before making their move. Dynamic games can be repeated a number of times. This enables the competitors to know and work with the result of previous rounds.

Some models describe only the behaviour of two companies in the studied market (duopoly). The firms can choose to corporate or compete. When competing, firms in a duopolistic market usually resort to non-price competition to achieve corporate goals such as profit-maximisation and increase in market share. The firms may have information about the decisions of their rival. This influences the strategies they adopt in achieving their goals. Others describe several companies with the same level of power (cartel). Others also assume that one of the companies has a dominant position in the market.

2.5. Empirical Literature
Several projects have been conducted in the area of Game Theory. Different methodologies have also been employed on such studies.

Araujo (2012) conducted a research on capital budgeting under competitive markets in the telecommunications industry in Portugal. The study focused on the economic feasibility of Fiber-to-the-Home networks using latest techniques. The methodological approach included the use of Game Theory, capital budgeting algorithms with real options and Monte Carlo simulations. These were adopted to analyse project risk. Results showed that with the use of Game Theory, Capital Budgeting Algorithm and Monte Carlo Simulation, firms in the telecommunications industry can evaluate the economic potential of their projected networks.

Guldmann and Kucukmehmetoglu (2002) analysed water resource issues with Game Theory taking Tigris and Euphrates rivers as a case study. They adopted a linear programming model that allocated water resources among agricultural and urban uses of Turkey, Syria, and Iraq. Cooperative game theory concepts such as Core and Shapley value were used to determine stable water allocations.

Gkonis and Psaraftis (2007) adopted a Game Theoretic approach to the analysis of the investment rules and competitive patterns in LNG (Liquefied Natural Gas) shipping market. The research revealed some useful insights. It demonstrated that it is important to consider the reaction of other players when making a decision. It also revealed that it is critical to gain information about the types of competitors in a game.

Fernandez (2008) applied the concept of Game Theory to examining the sharing of penalties and rewards in projects. The study examines how to divide the total reward and penalty among project activities. A game theoretic approach is used to determine a set of stable allocations of the total penalty and reward. Findings showed that the total reward for the project does not need to be equal to the summation of the rewards for the subprojects.

Mattos, Vieira, Schmitz, and Alencar (2014) used Game Theory to analyse incremental funding method in Software projects. The Incremental Funding Method (IFM) is a popular technique for maximising the financial return of software projects in a monopolistic market structure. Gambit, a tool for solving finite games, was used in determining all Nash equilibria. The findings of the research demonstrated that in a competitive market the equilibrium solutions enable competitors to choose the optimal monopolistic implementation order.

Serghini (2003) analyses water resource issues in multipurpose dam projects in Morocco through a Game Theoretic approach. He adopted the methodology of the FDC (Fully Distributed Costs) allocation rules, and applied it with the Shapley value and the Nucleolus. The multipurpose use of water in Morocco includes hydroelectric power, irrigation, and urban supply. There are usually conflicts over tariffs between the ministries that manage the entire infrastructure. The author examined two methods for allocating the costs: the FDC methods and the main apportioning methods.

Achugamonu, Inyama and Onuoha (2012) examined the objective project optimisation for the Nigerian Telecommunications Sector with the use of Game Theory. MTN Nigeria
and Globacom Nigeria were used as the case studies. Primary and Secondary data were used for the investigation. The methodology employed was linear programming. Findings showed that competitors in the Nigerian telecommunications industry should employ Game Theory in taking actions for optimal results in achieving company objectives. It was recommended that MTN Nigeria and Globacom Nigeria use the results of the study in allocating resources and planning to ensure better results.

Oziegbe (2011) applied the concept of Game Theory to examining business strategy in undeveloped countries adopting Nigeria as a case study. The methodology employed was a strategic form of a game. Results showed that game theory is an essential tool for examining the strategies of managers when determined by the actions of a competitor.

Lippai and Heaney (2000) applied Game Theory to present a method for determining efficient and equitable impact fees for urban water systems for each user, based on the type of demand on the system. The study assessed the significance of assuring a fair assessment of impact fees for an urban water supply system. The methodology used was cooperative game theory allocation solutions. The conclusion of the research was that the cost of the project needs to be equitably allocated among all existing and new users.

Fudenberg and Levine (1988) applied the concept of Game Theory to open-loop and closed-loop equilibria in dynamic games in the case of many players. Non atomic games in which the pay-off of a player is not influenced by the actions of a single rival were examined. The limits of a finite game approaching an atomic game were also examined. The objective was to demonstrate that equilibria in the non atomic game are approximately the same as those in the approaching finite game and that the limit of equilibria is an equilibrium and every sequence of equilibria has a limit. Two-period models were adopted for the analysis. Findings showed that as strategic possibilities increase with the game and the set of closed-loop equilibria expands, it becomes more difficult to guarantee that closed-loop equilibria are close to the open-loop equilibria.

Montero (2005) examined the behaviour of competitors when they bargain. The results indicated that altruism may be beneficial in bargaining when there is competition for bargaining partners. The results also indicate that the similar features of preferences that are beneficial in two-player bargaining can be detrimental when competition exists between bargaining partners. None of the studies conducted on the telecommunications industry have addressed the competition among GSM operating firms in Nigeria.

3. RESEARCH METHOD

The study employed secondary data, that is, data on the amount spent on television advertisement and radio advertisement by each GSM firm was collected. Data was sourced from Media Monitoring Services Limited and Internet. The period of the study covers from June 2014 to June 2015. The Linear Program Solver that is based on the efficient implementation of the modified simplex method was used for the computation.
of the results. The application is used for solving linear, integer, goal programming problems and for conducting sensitivity analysis.

3.1. Model Specification

Assuming two GSM operating firms A and B have M billion Naira and “N billion Naira respectively for adopting television advertisement and radio advertisement as strategies.

The Solution for firm A is represented by the following Linear Program Problem:

Maximize $\sum_{j=1}^{n} c_j x_j$

Subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m$

$x_j \geq 0, j = 1, 2, \ldots, n$

Where $x_j$ represents the strategy

The Solution for firm B is the Dual of the Solution for firm A. This is represented by the following Linear Program Problem:

Minimize $\sum_{i=1}^{m} b_i y_i$

Subject to $\sum_{i=1}^{m} y_i a_{ij} \geq c_j, j = 1, 2, \ldots, n$

$y_i \geq 0, i = 1, 2, \ldots, m$

Where $y_i$ represents the strategy

The competition between Nigerian GSM firms can be analysed through the prisoner’s dilemma game. Nigerian GSM companies produce products and services that are functionally the same. They can either cooperate or compete. If they both cooperate, they can charge monopoly prices; this would translate to having the same market share. However, they usually choose to compete, this is due to the fact that a critical goal of each firm is to maximize market share. In a bid to achieve this, each firm reduces the price of their product to achieve a higher market share. Examples of this strategy is GSM operating firms reducing their tariff for making calls and offering low prices for Internet services.

In any Game Theoretical Framework, linear programming algorithm has been found to be an effective tool in finding the optimal strategy of any firm in a competitive economy. According to Fabayo (2009), Linear Programming is a mathematical optimisation technique which is designed to determine the optimal allocation of scarce resources among competing products or activities. The optimal outcome can be maximum profit or minimum cost. Linear programming has been employed to determine advertising budgets to allocate resources to television, radio, billboard and newspaper advertisements.

4. PRESENTATION AND ANALYSIS OF RESULTS

The objective of this project is to determine the pay-off from television advertisement and radio advertisement for each GSM firm. To achieve this, yearly data on the expenditure on television advertisement and radio advertisement by each firm were
collected. The period of data collection was from June 2014 to June 2018. This information is presented in the table 1.

**Table 1:** Firms’ Expenditure on Television Advertisement and Radio Advertisement from June 2014 to June 2018

<table>
<thead>
<tr>
<th>Firm</th>
<th>Expenditure on Television (Billion Naira)</th>
<th>Expenditure on Radio (Billion Naira)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTN</td>
<td>1.8</td>
<td>0.9</td>
</tr>
<tr>
<td>Globacom</td>
<td>0.7</td>
<td>0.4</td>
</tr>
<tr>
<td>Airtel</td>
<td>1.2</td>
<td>0.4</td>
</tr>
<tr>
<td>9Naija</td>
<td>1.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Source: Media Monitoring Services Ltd (MMSL)

**Table 2:** Approximated Total Expenditure on Television Advertisement and Radio Advertisement from June 2014 to June 2018

<table>
<thead>
<tr>
<th>Firm</th>
<th>Expenditure on Television (Billion Naira)</th>
<th>Expenditure on Radio (Billion Naira)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MTN</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>Globacom</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Airtel</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>9Naija</td>
<td>11</td>
<td>5</td>
</tr>
</tbody>
</table>

Source: Author’s computation (2019)

Tables 1 and 2 can be used to describe a two-person duopolistic game. This game can be solved by employing the Simplex Method of Linear Programming and the Linear Program Solver.

### 4.1. Competition between MTN and Globacom

Let $x_1$ be the probability representing Television Advertisement and $x_2$ the probability representing Radio Advertisement. Let the value of the game for MTN be represented by $V_M$. Using the entries in Table 1 and 2, the study formulates the following Linear Programming for MTN Solution.

#### 4.1.1. MTN Solution
Maximize \( x_0 = x_1 + x_2 \)
\[
\text{Subject to } 7x_1 + 4x_2 \leq 1
\]
\[
18x_1 + 9x_2 \leq 1
\]
\( x_1, x_2 \geq 0 \)

Solving the Linear Programming Problem with the Simplex Method and the Linear Program Solver, it becomes;
\[
x_0 = 1/9, \ x_1 = 0 \text{ and } x_2 = 1/9
\]
Since we are maximizing \( V_M = 1/x_0 = 9 \)
The required proportions (probability) is
\[
X_1 = x_1/x_0 = 0 \times 9 = 0
\]
\[
X_2 = x_2/x_0 = 1/9 \times 9 = 1
\]

4.1.2. Globacom Solution

Globacom’s minimisation problem is the dual of MTN’s maximisation problem. Using the Duality Theory, the conversion is as follows.

Let \( y_1 \) be the probability representing Television Advertisement. Let \( y_2 \) be the probability representing Radio Advertisement and \( V_G \) represent the value of the game for Globacom. Using the Duality Theory we have

Minimize \( y_0 = y_1 + y_2 \)
\[
\text{Subject to } 7y_1 + 18y_2 \geq 1
\]
\[
4y_1 + 9y_2 \geq 1
\]
\( y_1, y_2 \geq 0 \)

Solving the Linear Programming Problem with the Simplex Method and Linear Program Solver, we have:
\[
y_0 = 1/9, \ y_1 = 0 \text{ and } y_2 = 1/9
\]
\[
V_G = 1/y_0 = 9/1 = 9
\]
The required proportions (probability) is
\[
Y_1 = y_1/y_0 = 0 \times 9 = 0
\]
\[
Y_2 = y_2/y_0 = 1/9 \times 9/1 = 1
\]
The results show that when competing with Globacom, for MTN to maintain its position as the leader in the Nigerian GSM Industry, it should invest 0% of the M billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will yield a pay-off of M billion Naira. Thus the optimal allocation of resources for MTN is

Television Advertisement = 0%
Radio Advertisement = 100%
The findings also show that for Globacom to change the structure of the Nigerian GSM Industry by attaining the position of the leader in the industry and thus gaining the largest market share, it should allocate 0% of the N billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will produce a pay-off of 9M billion Naira. Thus the optimal allocation of resources for Globacom is

Television Advertisement = 0%
Radio Advertisement = 100%

4.2. Competition between MTN and Airtel

Let \( a_1 \) be the probability representing Television Advertisement and \( a_2 \) the probability representing Radio Advertisement. Let the value of the game for MTN be represented by \( V_M \).

Using the entries in Table 1 and 2 we formulate the following Linear Programming for MTN Solution.

4.2.1. MTN Solution

Maximize \( \alpha_0 = a_1 + a_2 \)

Subject to

\[ 12a_1 + 4a_2 \leq 1 \]
\[ 18a_1 + 9a_2 \leq 1 \]
\[ a_1, a_2 \geq 0 \]

Solving the Linear Programming Problem, we have:

\( \alpha_0 = 1/9, a_1 = 0 \) and \( a_2 = 1/9 \)

Since we are maximizing \( V_M = 1/\alpha_0 = 9 \)

The required proportions (probability) is

\( A_1 = a_1/\alpha_0 = 0 \times 9 = 0 \)
\( A_2 = a_2/\alpha_0 = 1/9 \times 9 = 1 \)

4.2.2. Airtel Solution

Airtel’s minimisation problem is the dual of MTN’s maximisation problem. Using the Duality Theory, the conversion is as follows.

Let \( b_1 \) be the probability representing Television Advertisement. Let \( b_2 \) be the probability representing Radio Advertisement and \( V_A \) represent the value of the game for Airtel. Using the Duality Theory we have

Minimize \( \beta_0 = b_1 + b_2 \)

Subject to

\[ 12b_1 + 18b_2 \geq 1 \]
\[ 4b_1 + 9b_2 \geq 1 \]
\[ b_1, b_2 \geq 0 \]
Solving the Linear Programming Problem, we have:

\[ b_0 = \frac{1}{9}, \ b_1 = 0 \ \text{and} \ b_2 = \frac{1}{9} \]

VA = \frac{1}{b_0} = \frac{1}{9} \times 9 = 9

The required proportions (probability) is

B1 = \frac{b_1}{b_0} = 0 \times 9 = 0

B2 = \frac{b_2}{b_0} = \frac{1}{9} \times 9 = 1

The results show that when competing with Airtel, for MTN to maintain its position as the leader in the Industry, it should invest 0% of the M billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will yield a pay-off of M billion Naira. Thus the optimal allocation of resources for MTN is

Television Advertisement = 0%
Radio Advertisement = 100%

The findings also show that for Airtel to change the structure of the Industry by attaining the position of the leader and thus acquiring the largest market share, it should allocate 0% of the M billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will produce a pay-off of M billion Naira. Thus the optimal allocation of resources for Airtel is

Television Advertisement = 0%
Radio Advertisement = 100%

4.3. Competition between MTN and 9Naija

Let \( c_1 \) be the probability representing Television Advertisement and \( c_2 \) the probability representing Radio Advertisement. Let the value of the game for MTN be represented by \( V_M \). Using the entries in Table 1 and 4.2 we formulate the following Linear Programming for MTN Solution.

4.3.1. MTN Solution

Maximize \( c_0 = c_1 + c_2 \)
Subject to
\[ 11c_1 + 5c_2 \leq 1 \]
\[ 18c_1 + 9c_2 \leq 1 \]
\[ c_1, c_2 \geq 0 \]

Solving the Linear Programming Problem with the Simplex Method, we have:

\( c_0 = \frac{1}{9}, \ c_1 = 0 \ \text{and} \ c_2 = \frac{1}{9} \)

Since we are maximizing \( V_M = \frac{1}{c_0} = 9 \)

The required proportions (probability) is

\( C_1 = \frac{c_1}{c_0} = 0 \times 9 = 0 \)

\( C_2 = \frac{c_2}{c_0} = \frac{1}{9} \times 9 = 1 \)
4.3.2. **9Naija Solution**

9Naija’s minimization problem is the dual of MTN’s maximization problem. Using the Duality Theory, the conversion is as follows.

Let $d_1$ be the probability representing Television Advertisement. Let $d_2$ be the probability representing Radio Advertisement and $V_E$ represent the value of the game for 9Naija’. Using the Duality Theory we have

Minimize $d_0 = d_1 + d_2$

Subject to

$11d_1 + 18d_2 \geq 1$

$5d_1 + 9d_2 \geq 1$

$d_1, d_2 \geq 0$

Solving the Linear Programming Problem, we have:

$d_0 = 1/9$, $d_1 = 0$ and $d_2 = 1/9$

$V_E = 1/d_0 = 1 \times 9 = 9$

The required proportions (probability) is

$D1 = d_1/d_0 = 0 \times 9 = 0$

$D2 = d_2/d_0 = 1/9 \times 9 = 1$

The results revealed that when competing with 9Naija’, for MTN to maintain its position as the leader in the Nigerian GSM Industry, it should invest 0% of the M billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will yield a pay-off of 9M billion Naira. Thus the optimal allocation of resources for MTN is

Television Advertisement = 0%

Radio Advertisement = 100%

The study also showed that for 9Naija’ to attain the position of the leader in the Industry and thus gain the largest market share, it should allocate 0% of the K billion Naira funds available to the firm into Television Advertisement and 100% into Radio Advertisement. This will produce a pay-off of 9K billion Naira. Thus the optimal allocation of resources for 9Naija’ is

Television Advertisement = 0%

Radio Advertisement = 100%

**5. CONCLUSIONS AND RECOMMENDATIONS**

Game Theory can be a significant tool for GSM firms for optimising resource allocation between strategies. Firms usually face the challenge of deciding on the best allocation of scarce resources on competing needs in a bid to gain the largest market share. An inefficient allocation of resources leads to waste and incurring losses. This can be prevented by adopting a Game theoretic approach.
Game Theory enables firms to optimise the allocation of limited resources among various strategies in gaining the largest market share. It also enables firms to derive the allocation of resources that will yield an optimal benefit.

Nigerian GSM firms can determine the pay-off from the strategies available to them with the use of Game Theory. Such organisations are often concerned about the pay-off from their actions. They intend to know the benefits obtainable from their strategies before committing resources to such decision. This enables them to determine whether their strategies are profitable or inefficient. This is possible with the use of game theory. Game theory can serve as a tool for determining the pay-off from the strategies and firms are considering in acquiring the largest market share. Thus game theory can enable firms to evaluate the effectiveness of the strategies available to them.

5.1. Recommendations

Based on the results of this research, it is recommended as follows:

i. GSM firms should employ Game Theory in determining the optimal strategy from television advertisement and radio advertisement in gaining the largest market share.

ii. GSM firms should use Game Theory in determining the pay-off from the optimal strategies they adopt.

iii. GSM firms should employ Game Theory in optimising resource allocation between television advertisement and radio advertisement.

iv. GSM firms in the Nigerian Telecommunications Industry should consider the strategies of their competitors in determining the best decision to adopt.

References


